Table 2 Maximum and mean wind speed shears and frequency of occurrence of shears >0.1 s^{-1} as a function of layer, distance, and sustained wind speed

Height, m	Int. f	Obs.	Mean *WS* Ref. m	Mean *WS* range ms^{-1}	*WS* Shear, s^{-1} Max	*WS* Shear, s^{-1} Mean	*WS* Shear >0.1 s^{-1} f	*WS* Shear >0.1 s^{-1} %
150	15	750	120	0<5	0.040	0.014	0	0
	16	800		5<10	0.143	0.014	9	0.23
	41	2050		10<18	0.160	0.028	57	1.44
	7	350		18<33	0.130	0.030	6	0.15
120								
	6	300	90	0< 5	0.050	0.019	0	0
	14	700		5<10	0.107	0.029	3	0.10
	38	1900		10<18	0.173	0.035	65	2.06
	5	250		18<33	0.127	0.035	7	0.22
90								
	5	250	60	0< 5	0.050	0.016	0	0
	23	1150		5<10	0.143	0.037	33	1.05
	32	1600		10<18	0.327	0.042	132	4.19
	3	150		18<33	0.177	0.059	19	0.60
60								
	16	800	30	0< 5	0.100	0.039	0	0
	41	2050		5<10	0.200	0.041	245	6.20
	22	1100		10<18	0.387	0.062	357	9.04
	0	0		18<33	0	0	0	0
30								
	22	1100	18T	0< 5	0.158	0.059	133	3.37
	45	2250		5<10	0.783	0.102	874	22.13
	12	600		10<18	0.792	0.135	271	6.86
	0	0		18<33	0	0	0	0
18								
	22	1100	18S	0< 5	0.227	0.076	283	7.16
	45	2250		5<10	0.693	0.168	1638	41.46
3	12	600		10<18	0.713	0.310	544	13.77
	0	0		18<33	0	0	0	0
Distance, m 18	22	1100	18T	0< 5	0.122	0.023	8	0.20
	45	2250		5<10	0.356	0.075	565	14.30
	12	600		10<18	0.678	0.136	298	7.54
	0	0		18<33	0	0	0	0

i.e., for 3-30 m 0.7 and 0.14 s^{-1}, for 30-90 m 0.4 and 0.04 s^{-1}, and for 90-150 m 0.2 and 0.03 s^{-1}; 2) percentage frequency of occurrence of magnitudes greater than 0.1 s^{-1} decreases by a factor of 0.5 from the lowest layer to the highest, i.e., 62% for 3-18, 32% for 18-30, 15% for 30-60, 6% for 60-90, 2.4% for 90-120, and 1.8% for 120-150 m layer; 3) maximum and mean horizontal shear magnitudes at 18 m are slightly less than the vertical counterparts below 30 m and have a 22% frequency of occurrence for shears greater than 0.1 s^{-1}; and 4) significant number of occurrences of wind speed shears greater than 0.1 s^{-1} below 30 m during low (0<5 ms^{-1}) sustained speed questions the generally accepted belief that low wind speeds and subsequent light shears present little or no hazards to aviation safety.

This study certainly lends support to the ideas that short-term wind measurement accuracy is vital in wind shear detection and that the need for information on low-level shear is most important over the lowest 150 m of the Earth's atmosphere—may be even more important over the lowest 60 m.

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Application of Unsteady Airfoil Theory to Rotary Wings

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A PREVIOUS Note[1] pointed out that unsteady airfoil theory is being used incorrectly in almost all major helicopter loads analyses and also in some aeroelastic stability analyses. The difficulty lies in relating the variables used in unsteady airfoil theory to the variables used in describing the motion of a rotary wing. Reference 1 presented an attempt to identify correctly the relationship between the two sets of variables and to apply the resulting theory to an articulated

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rotor blade having rigid-flap and rigid-pitch motions in forward flight. The purpose of this Note is to clarify the recent background on the application of unsteady airfoil theory to rotary wings and to elucidate the discussion presented in Ref. 1.

The relationship between the variables used in unsteady airfoil theory and the variables required to describe the more complex motion of a rotary wing was correctly identified and discussed by the authors in Ref. 2. Furthermore, the procedure for systematically establishing the relationship presented in Ref. 2 was subsequently applied to the formulation of nonlinear aeroelastic equations of both horizontal[3] and vertical axis[4] wind-turbine blades. It is believed that the procedure presented in Ref. 2 is not only clearer and more rigorous than the one presented in Ref. 1 but, more importantly, provides the basis for readily applying it to any of the usual mathematical models employed to represent rotary wings in both hover and forward flight.

The application of unsteady airfoil theory to rotary wings may be conveniently regarded as consisting of four steps. The first step is the selection of an appropriate unsteady airfoil theory. To this end, Ref. 2 employed the theory of Greenberg,[5] who extended the Theodorsen theory for a thin, two-dimensional airfoil undergoing unsteady motion in an incompressible flow to account for a time-varying freestream velocity. From Ref. 5, the expression for lift and pitching moment, specialized to the case of interest here, may be written as

$$L=\pi\rho b^2[(\dot{h}+V\epsilon)^{\dot{}}-ba\ddot{\epsilon}]+2\pi\rho VbC[\dot{h}+V\epsilon+b(\tfrac{1}{2}-a)\dot{\epsilon}] \tag{1a}$$

$$M=\pi\rho b^2[ba(\dot{h}+V\epsilon)^{\dot{}}-V\frac{b}{2}\dot{\epsilon}-b^2(\tfrac{1}{8}+a^2)\ddot{\epsilon}] +2\pi\rho Vb^2(a+\tfrac{1}{2})C[(\dot{h}+V\epsilon)+b(\tfrac{1}{2}-a)\dot{\epsilon}] \tag{1b}$$

It should be remarked that the velocity V is a function of time and that the lift deficiency function C must account for the expansion and compression of the shed wake vorticity. The above equations are the same as the final lift and moment expressions given in Ref. 1 except that ϵ and V are used here instead of α and U to be consistent with the notation of Ref. 2.

The second step is to resolve the velocity V, which is the resultant of aerodynamic and dynamic velocities at a point on the elastic axis, into three components U_R, U_T, and U_P (radial, tangential, and perpendicular components). Because the quantity $\dot{\epsilon}$ is the angular velocity of a blade section about the deformed elastic axis, it is appropriate to express U_R, U_T, and U_P with respect to the coordinate system of the deformed blade.

The third step is to express L and M in terms of U_R, U_T, U_P, and $\dot{\epsilon}$. To this end, let us identify the significance of the terms in Eq. (1a). It can be seen that the expressions in the first and second set of square brackets of Eq. (1a) are the downward acceleration of the midchord point of the airfoil and the downward velocity of the three-quarter chord point of the airfoil, respectively. Since U_P is the perpendicular component relative to a point on the elastic axis, one immediately obtains the identity

$$\dot{h}+V\epsilon=-U_P \tag{2}$$

Substituting Eq. (2) into Eqs. (1), yields

$$L=\pi\rho b^2[-\dot{U}_P-ba\ddot{\epsilon}]+2\pi\rho VbC[-U_P+b(\tfrac{1}{2}-a)\dot{\epsilon}] \tag{3a}$$

$$M=\pi\rho b^2[-ba\dot{U}_P-\frac{Vb}{2}\dot{\epsilon}-b^2(\tfrac{1}{8}+a^2)\ddot{\epsilon}] +2\pi\rho Vb^2(a+\tfrac{1}{2})C[-U_P+b(\tfrac{1}{2}-a)\dot{\epsilon}] \tag{3b}$$

where

$$V=\sqrt{U_R{}^2+U_T{}^2+U_P{}^2} \tag{4}$$

The fourth step is to derive explicit expressions for U_R, U_T, U_P, and ϵ in terms of forward flight velocity, induced flow, rotor rotational speed, blade motion variables, control inputs, etc. These expressions depend on the type of mathematical model used to represent the rotary wing. When these expressions are substituted into Eqs. (2) and (4), the resulting equations represent the correct relationship of the variables of unsteady airfoil theory and the variables of rotary wing theory.

In most of the literature, the common errors made are in identifying the correct relationship between the unsteady airfoil theory variables and the rotary wing variables given by Eq. (2) and in deriving the expressions for U_R, U_T, U_P, and $\dot{\epsilon}$. These aspects were discussed in Ref. 2.

Reference 1 mentioned the relationship between the two sets of variables given by Eq. (2) and presented the correct expressions for U_P‡ and $\dot{\epsilon}$ and for a simple illustrative example. Reference 2 also derived the relationship given by Eq. (2) and presented the expressions for L and M given by Eqs. (3) and (4) for the case in which the elastic axis is at the quarter chord point ($a=-\tfrac{1}{2}$) and $C=1$. Additionally, Ref. 2 rigorously derived the explicit expressions for U_R, U_T, U_P, and $\dot{\epsilon}$ in terms of forward flight velocity, blade motion variables, etc. These expressions are valid for both linear and second-degree nonlinear formulations of a flexible rotary wing in both hover and forward flight. Furthermore, these expressions can be specialized and applied to the case of a rotary wing modeled as either a spring-restrained or fully articulated rigid blade. To illustrate the application of these expressions, consider the example in Ref. 1. For this case, the expressions for $(\dot{h}+V\epsilon)$ and $\dot{\epsilon}$ can be obtained by specializing the expressions for U_P and $\dot{\epsilon}$ given in Ref. 2 and are

$$\dot{h}+V\epsilon=-U_P=(\Omega r+\mu\Omega R\sin\psi)\theta-(\lambda\Omega R+r\dot{\beta}+\mu\Omega R\beta\cos\psi) \tag{5}$$

$$\dot{\epsilon}=\dot{\theta}+\Omega\beta \tag{6}$$

which are in agreement with the corresponding ones in Ref. 1.

In summary, the authors of this Note identified and discussed the correct application of unsteady airfoil theory to rotary wings prior to the publication of Ref. 1. The procedure of Ref. 2 is thought to be clearer, more rigorous, and more general than the one presented in Ref. 1.

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